Statistical and Low Temperature Physics (PHYS393)

# 7. Superconductivity

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1. Resistance, magnetic field and heat capacity observations.

2. Explanation using macroscopic wavefunction.

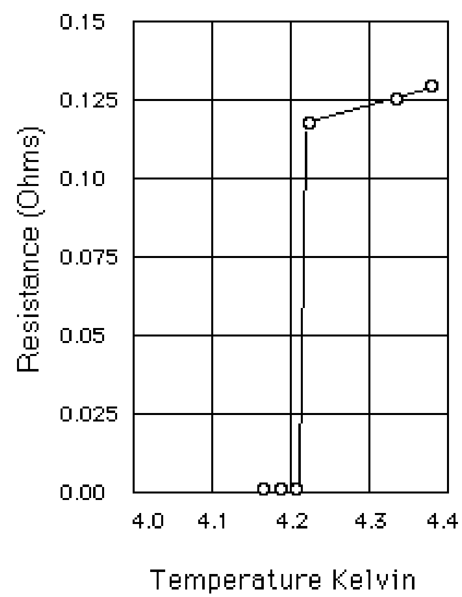
3. Quantised vortices.

4. Cooper pairs.

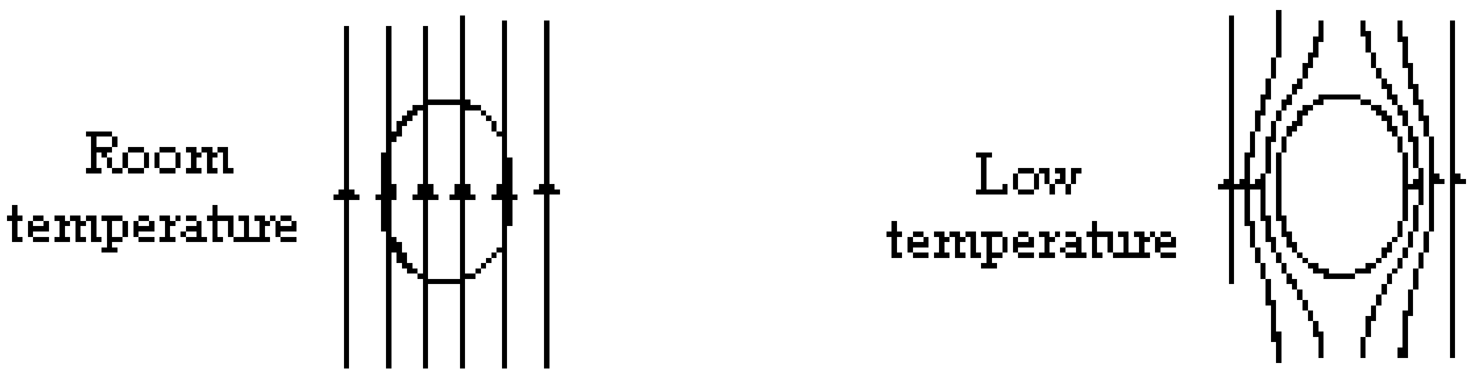
5. Applications: trains, accelerators, ...

Metals conduct electricity. Normally, there is always some resistance, however small.

In some materials, this resistance suddenly falls to zero below a certain temperature. In 1911, Kamerlingh Onnes discovered that this happened with mercury below 4.2 K



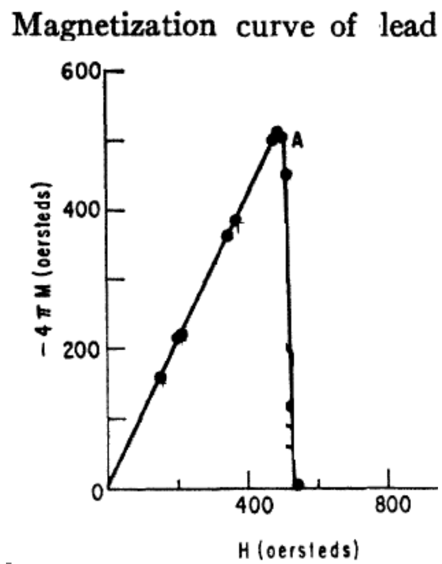
When the resistance drops to zero, the superconductor all expels all magnetic field from its body.




`http://www.materia.coppe.ufrj.br/sarra/artigos/artigo10114/index.html`

The field inside the body of a superconductor can be obtained by inserting it in a coil and measuring the induced voltage.

This graph shows the magnetisation of lead in liquid helium, plotted against the applied field.




Livingston, Physical Review, vol. 129 (1963), p. 1943

Below a certain critical field, the magnetisation is equal and opposite to the applied field. So the resultant field inside the superconductor is zero.

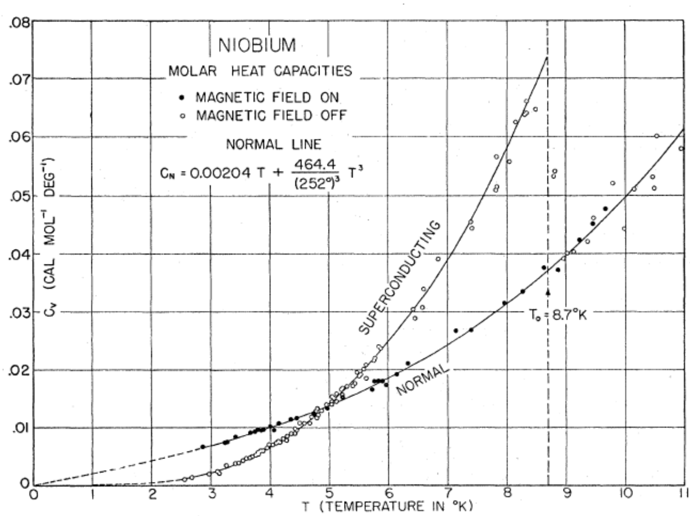
The expulsion of magnetic field from a superconductor is called is Meissner effect.

A striking demonstration is the levitation of a superconductor above a magnet.

Recall the heat capacity of a normal metal:

$$C_v = \gamma T + AT^3.$$

Measurements show that for supercondctors, this changes completely below the transition temperature. This graph is the result of measurement for the niobium metal.



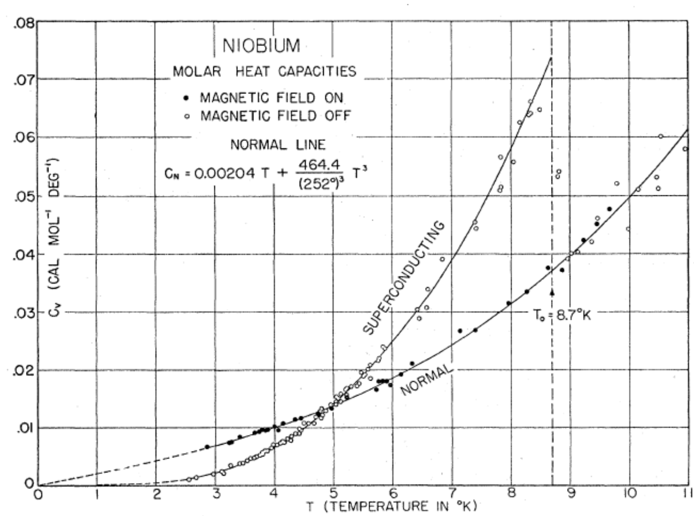

Brown, et al, Physical Review, vol. 92 (1953), p. 52

Niobium become superconducting below 9.5 K. It is possible to prevent it from becoming superconducting by applying a sufficiently large enough magnetic field.

We know from the Meissner effect that a niobium expels all magnetic field. However, if the field is strong enough, it can "force" its way into the superconductor. This destroys the superconductivity and returns the niobium to a normal conducting state - even if temperature is below 9.5 K.

Using this property, it is possible to select between the normal and the superconducting state.

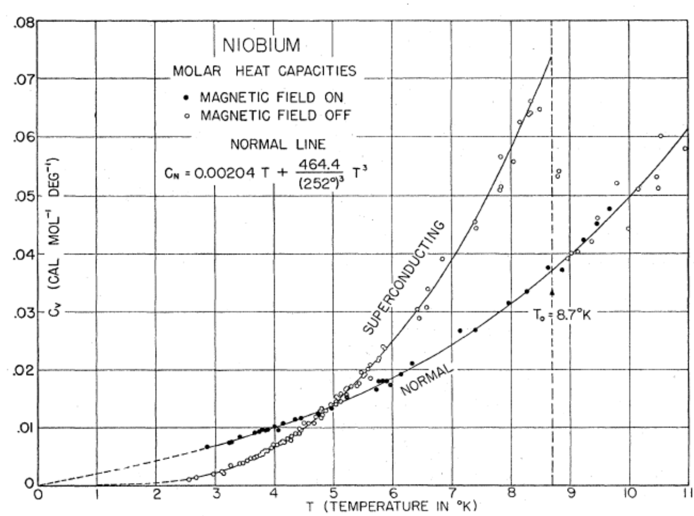
If we select the normal conducting state of niobium by applying a strong magnetic field, we would measure the curve labelled "normal."




This follows the "normal" behaviour of

$$C_v = \gamma T + AT^3.$$

If we do not apply any magnetic field, we get the superconducting state. Then we would get the curve labelled "superconducting."




If we plot log$C_v$ against 1/T for this curve, we would find a straight line. This means that the heat capacity is given by

$$C_v = a\,\mathrm{exp}\left(-\frac{b}{T}\right),$$

for some constants $a$ and $b$.

In 1937, Fritz London suggested that if the electrons in a superconductor somehow forms a macroscopic wavefunction.

Using this assumption, London was able to explain why a superconductor has zero resistance, and why it expels all magnetic field.

We have already seen how a macroscopic wavefunction can explain zero viscosity in a superfluid. A superfluid flow against other bodies would experience no resistance, unless the flow is energetic enough to cause an excitation to the next energy level.

Resistance to an electrical current normal comes from the electrons scattering of phonons (vibrations) and impurities.

If the electrons can somehow form a macroscopic wavefunction, then the same ideas for a superfluid would apply. The wavefunction would flow pass the phonons and impurities and not experience any resistance - unless the flow the energetic enough to excite the wavefunction to the next energy level.

This gives us the first part of the story. It took another 20 years to explain why a macroscopic wavefunction is possible. We shall return to this later.

We now try to answer why a superconductor expels magnetic field. Before that, we first need to appreciate why expulsion of the magnetic field is strange.

Suppose the resistance going to zero is the only change in a metal. Consider what happens if we now bring a magnetic to the metal.

The change in magnetic flux through the metal induces an electric current, according to Faraday's law.

According to Lenz's law, the current would flow in such a way as to produce a magnetic field of its own that opposes the incoming field.

In a metal with resistance, this induced current would quickly slow down to zero. The induced field becomes zero, and only the incoming field remains in the body of the metal.

If the metal has no resistance, the induced current continues to flow. The induced flux has to be opposite to the incoming flux. Therefore they cancel, and the field in the body becomes zero. In this way, the field is "expelled."

It looks like we have just “explained” the Meissner effect. However, let us now look at what happens if the magnet is already there before cooling.

We start with a normal metal with a magnetic field going through the body. Then we cool this down and the resistance falls to zero.

According to Faraday’s law, since there is no change in magnetic flux, no current is induced. So the original field from the magnet remains in the body.

In a real superconductor, we know from the Meissner effect that, even in this case, the magnetic field must be expelled.

This shows that there is something different about a superconductor that the familiar laws of electromagnetism cannot explain.

We shall now see how a macroscopic wavefunction, $\psi$, can explain the Meissner effect.

Recall the operator in quantum mechanics for momentum:

$$-i\hbar\frac{d\psi}{dx} = p_x\psi$$

where $p$ is the momentum $mv$.

In the presence of an electromagnetic field, this is changed to

$$-i\hbar\frac{d\psi}{dx} = (mv - qA)\psi$$

where $A$ is the vector potential and $q$ the charge of the particle.

`http://quantummechanics.ucsd.edu/ph130a/130_notes/node29.html`

Both equations are quantum mechanical postulates that have been shown to give correct results in physics.

In order to use the vector potential, lets review its meaning. It is defined by

$$\nabla \times \mathbf{A} = \mathbf{B},$$

where $\mathbf{B}$ is the magnetic field. This a bit similar to relation between the electric field and electric potential.

http://en.wikipedia.org/wiki/Magnetic_potential

For a qualitative understanding, the integral form of this equation is sufficient:

$$\int_C \mathbf{A}.d\mathbf{l} = \int_S \mathbf{B}.d\mathbf{S},$$

where the left integral is along any loop C, and and the right integral is over any surface S enclosed by the loop.

The right side of this equation is the magnetic flux $\Phi$,

$$\int_C \mathbf{A}.d\mathbf{l} = \int_S \mathbf{B}.d\mathbf{S}$$

and the left side is the line integral for magnetic potential.

If we make the following replacements: $\mathbf{A} \to \mathbf{B}$ and $\mathbf{B} \to \mathbf{J}$, where $\mathbf{J}$ is the current density, we get Ampere's law.

In the more familiar Ampere's law, the electric current is related to the integral of magnetic field over a loop round the current.

In the same way, the equation

$$\int_C \mathbf{A}.d\mathbf{l} = \Phi$$

tells us that magnetic flux is equal to the integral of vector potential over a loop round the flux.

Let us now return to the quantum mechanical equation:

$$-i\hbar\frac{d\psi}{dx} = (mv - qA)\psi.$$

Recall the wavefunction we used for superfluids:

$$\psi = e^{-i\phi(x)}$$

where $\phi(x)$ is the phase. Substituting into the equation, we get

$$\hbar\frac{d\phi}{dx} = mv - qA.$$

This relation along a straight line in $x$ can be extended in a simple way to any path or loop in 3D.

Consider a loop in a superconductor of length $L$ enclosing an area $S$. Integrating along this loop, we get

$$\hbar\Delta\phi = m\int_L \mathbf{v}.d\mathbf{l} - q\int_L \mathbf{A}.d\mathbf{l}.$$

$$\hbar \Delta \phi = m \int_L \mathbf{v}.d\mathbf{l} - q \int_L \mathbf{A}.d\mathbf{l}.$$

The phase change $\Delta\phi$ is zero or a multiple of $2\pi$, because the wavefunction returns to the same value after one loop.

The integral over $\mathbf{A}$ gives the magnetic flux $\Phi$.

The velocity $\mathbf{v}$ is related to the current density $\mathbf{J}$ by

$$\mathbf{J} = \rho q \mathbf{v},$$

where $\rho$ is the number density of the electrons. The above equation then becomes

$$\hbar \Delta \phi = \frac{m}{\rho q} \int_L \mathbf{J}.d\mathbf{l} - q\Phi.$$

Let us now see how this equation

$$\hbar\Delta\phi = \frac{m}{\rho q}\int_L \mathbf{J}.d\mathbf{l} - q\Phi.$$

can help us understand Meissner's effect.

For a simple lump of metal, the wavefunction would be continuous through the whole volume, so the phase change would be zero. The equation then simplifies to

$$\frac{m}{\rho q}\int_L \mathbf{J}.d\mathbf{l} = q\Phi.$$

We have now obtained a very important result:

**If there is a magnetic field in the macroscopic wavefunction, then is a there is an electric current.**

To see why this is special, consider Faraday's law again.

According to Faraday's law, a *change* in magnetic flux is required before a current can be induced.

For a macroscopic wavefunction, the very presence of the flux produces the current. No change in flux is needed!

Let us look at the case of transition to the superconducting state again. Previously, we have not been able to explain the expulsion of the field using Faraday's law.

We can now explain this assuming that a macroscopic wavefunction appears when the metal becomes superconducting, If there is a magnetic field in the metal, it would produce a current. This current would in turn produce a flux.

A more detailed reasoning would show that this wavefunction flux is in the opposite direction to the incoming flux.

Flux from the wavefunction, or superconducting, current would cancel some of the incoming flux.

The amount cancelled depends on the density of the electrons in the wavefunction. The higher the density, the larger the superconducting current, and more of the incoming flux would be cancelled.

For a uniform external field, this superconducting current would typically be circulating the metal. So it produces the greatest field at the centre, where more cancellation takes place.

For larger electron density, the region of cancellation is also larger. In a typical superconductor, there is sufficent density to expel the incoming field from most of the volume.

In practice, some field would penetrate to a depth of about 100 nm on the surface.

The reason for the penetration depth is that a current is needed to keep the field expelled.

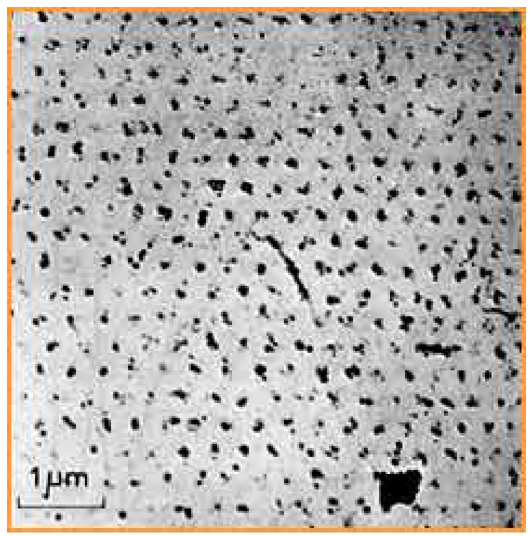
Recall that a field must be present in the macroscopic wavefunction in order to produce the current. As the field gets expelled from the center of the superconductor, the current at the center would also stop.

If the field is completely expelled from the metal, there would be no current at all in the metal. Then there would be no opposing flux to cancel the incoming flux. The external flux would come in again and start producing current.

For this reason, a balance would to be reached. The field would penetrate until a depth when there is sufficient current to keep the rest of the volume field free.

In the lectures on superfluid helium, we have seen that a macroscopic wavefunction can give rise to vortices that quantised.

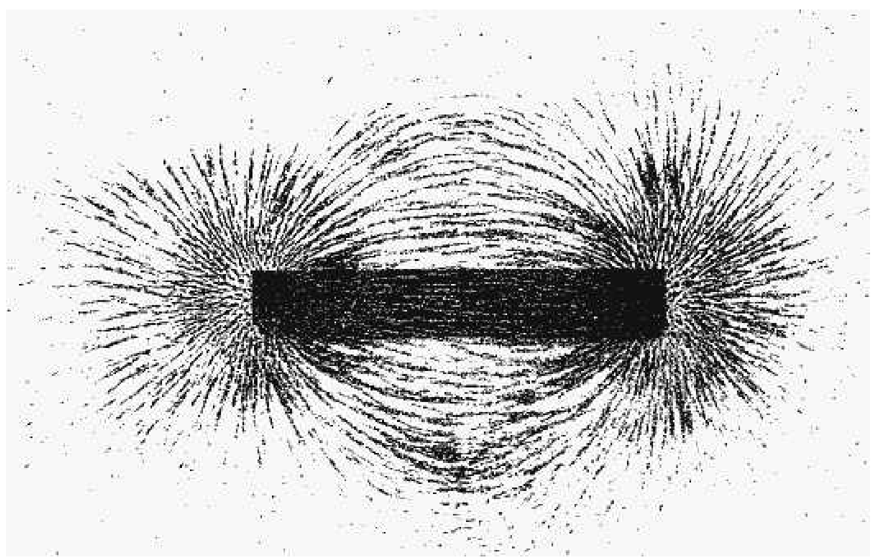
If the electrons in a superconductor also forms a macroscopic wavefunction, quantised vortices should also be possible in the electrons. This is indeed observed:



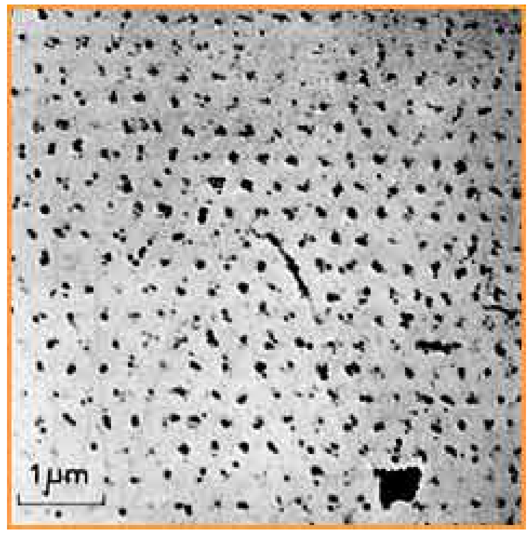

Essmann and Trauble, Physics Letters 24A, 526 (1967)

The method used to observe vortices is similar to the method for observing magnetic field lines in school.

Sprinkle some iron filings on a piece of paper, place a magnet underneath, tap the paper gently, and this is what you would see:



Likewise, Essmann and Trauble sprinkled some cobalt powder on a Lead-Indium alloy. This is what they saw under an electron microscope:




The cobalt powder collected at the centres of the vortices, where magnetic fields are strongest.

A nice gallery of superconducting vortices can be found here:
http://www.fys.uio.no/super/vortex/index.html

The existence of vortices is in fact not consistent with Meissner's effect.

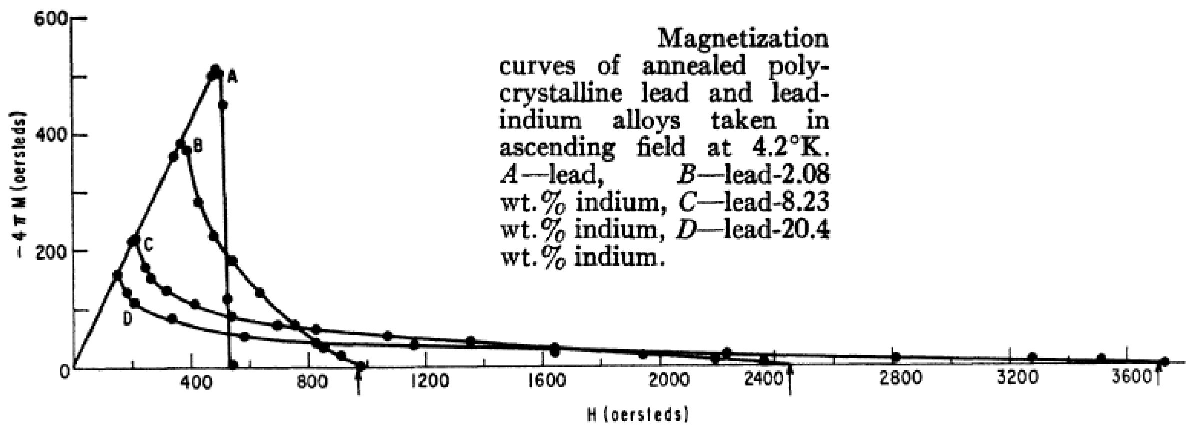
We have learnt that when a metal becomes superconducting, it expels all magnetic field (except for some near its surface).

A vortex in a superconductor is a circulating current. This must produce a magnetic field in the superconductor. This contradicts the Meissner's effect.

It turns out that the Meissner's effect is only true for some metals - mainly pure metals. These are called Type I superconductors.

For alloys and other materials, it is possible for magnetic field to penetrate the body of the superconductor to some extent.

This shows the magnetisation of Lead alloy with different amount of Indium:



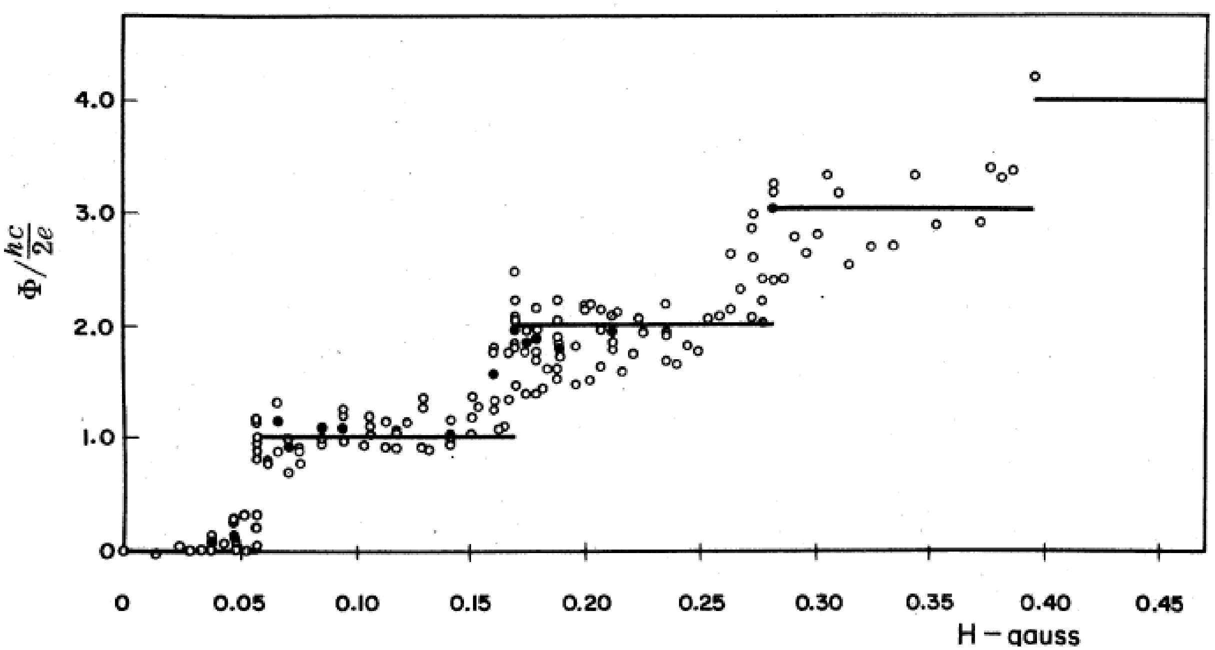

Magnetization curves of annealed polycrystalline lead and lead-indium alloys taken in ascending field at 4.2°K. *A*—lead, *B*—lead-2.08 wt.% indium, *C*—lead-8.23 wt.% indium, *D*—lead-20.4 wt.% indium.

Below a certain critical field, the magnetisation (e.g. OB) is strong enough to cancel the applied field.

For higher field, the magnetisation decreases (e.g. curve to the right of B). It is not enough to cancel the applied field, which then penetrates the superconductor. These are called type II superconductors.

We have seen that vortices of electrons do exist in a superconductor. Lets now look at whether they are quantised.

If the current around a vortex is quantised, so is the magnetic flux produced. This can be measured, the has indeed been found to be quantised.




Deaver and Fairbank, Physical Review Letters, vol. 7 (1961) p. 43

Deaver and Fairbank measured the flux through a long, thin tube made of Tin:

1. Apply a magnetic field to the tube.

2. Cool below the 3.7 K transition temperature.

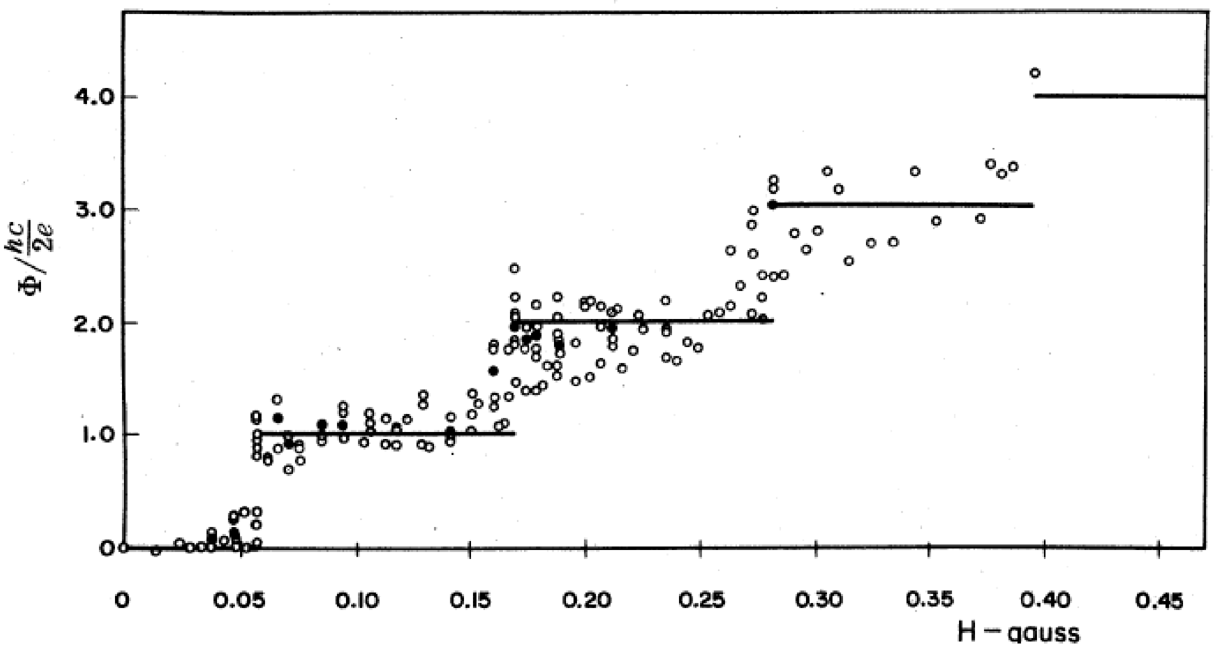
3. Move the tube up and down rapidly.

4. Place a coil near the end of the tube.

5. Measure the voltage induced in the coil.

6. Obtain the flux from the voltage.

The measured flux is plotted against applied field:




The steps show that the possible flux through the tube is indeed quantised. The magnitude of each step is

$$\Phi = \frac{h}{2e}.$$

In the case of the superfluid, no magnetic field is involved. So we still need to understand how vortex arise in the superconductor.

Recall the relation between flux and current in a macroscopic wavefunction:

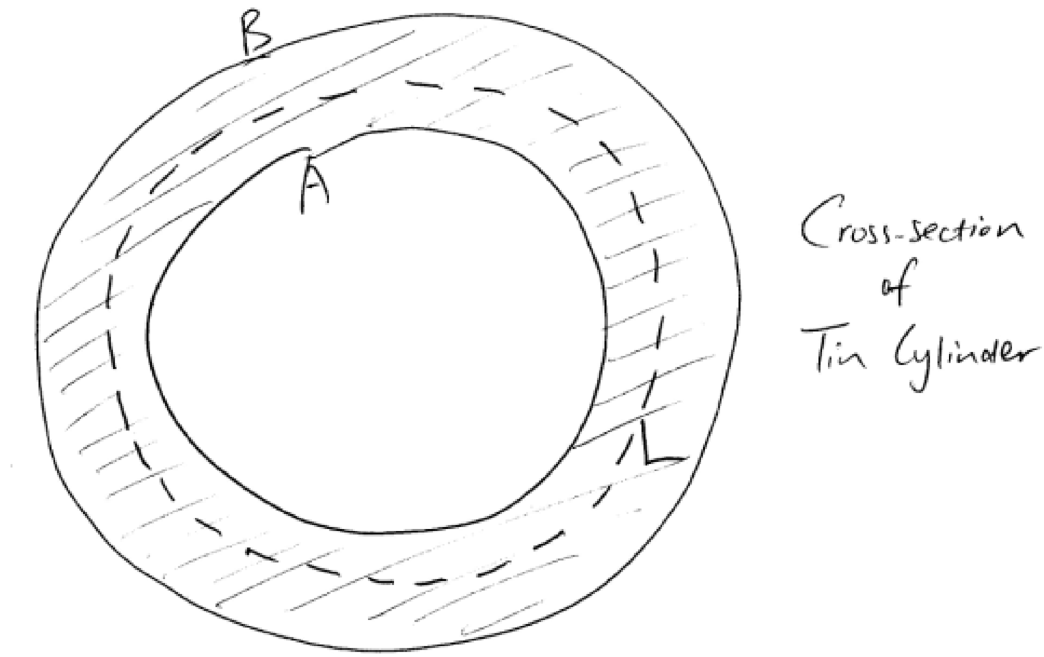
$$\hbar\Delta\phi = \frac{m}{\rho q}\int_L \mathbf{J}.d\mathbf{l} - q\Phi.$$

The Tin tube is a solid with a hole through it. The wavefunction is no longer continuous over the whole volume, so phase change around the tube does not have to be zero:

$$2n\pi\hbar = \frac{m}{\rho q}\int_L \mathbf{J}.d\mathbf{l} - q\Phi.$$

The equation is true for any loop $L$ in the wavefunction. It is possible to choose the loop in such that the integral over current $\mathbf{J}$ is zero.

This figure shows the cross-section of the Tin tube. We are interested in the flux through the hollow.




When this is superconducting, current is only possible very near the surfaces A and B, within the penetration depth. Further in the bulk, there is no current because there is no field, since all field is expelled.

So if we choose the loop $L$ away from either surfaces, then the current density along $L$ would be zero. The equation

$$2n\pi\hbar = \frac{m}{\rho q}\int_L \mathbf{J}.d\mathbf{l} - q\Phi.$$

then becomes

$$2n\pi\hbar = q\Phi.$$

The "-" sign can be left out if we are only interested in the magnitudes. Since $q$ is the charge of an electron, the flux is

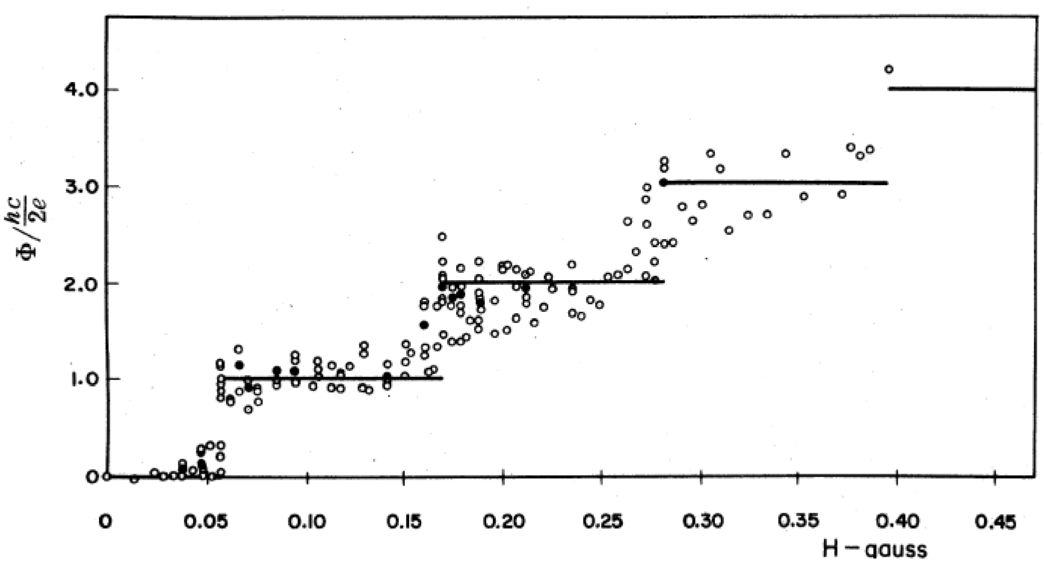
$$\Phi = \frac{nh}{e}.$$

This means that one quantum step is $h/e$.

We have just found a problem.

Using the macroscopic wavefunction, we have found that the flux is quantised in steps of $h/e$.

The measurement results tell us that the flux is quantised in steps of $h/2e$.



Notice the difference:

Theory predicts $h/e$. Measurement gives $h/2e$.

This means that something must be wrong with the theory. It seems to suggest that, instead of a charge of $e$, the particle should have a charge of $2e$.

This is one of the evidence to suggest that the electrons might somehow be moving in pairs.

Recall that the heat capacity of a superconducting metal is given by

$$C_v = a \,\mathrm{exp}\left(-\frac{b}{T}\right).$$

With some imagination, we may see that this looks rather like a Boltzmann factor. Lets make up something like this:

$$C_v = a \,\mathrm{exp}\left(-\frac{\Delta}{k_B T}\right)$$

where $b$ is replaced by $\Delta / k_B$.

This is the kind of expression we would get if we have two levels separated by the energy $\Delta$.

If two electrons do come in a pair, perhaps $\Delta$ is the energy needed to break up the pair?

If electrons repel each other, how can they attract to form a pair?

The clue came from the isotope effect. In 1950, the superconducting transition temperature of Mercury was found to be different for different isotopes of Mercury.

Transition temperatures.

| Sample | Average mass number | $T_0$(°K) |
|---|---|---|
| 1 | 203.4 | 4.126 |
| 2 | 202.0 | 4.143 |
| 3 | 200.7 | 4.150 |
| 4 | 199.7 | 4.161 |

Reynolds, et al, Physical Review, vol. 78 (1950) p. 487

The only difference between isotopes is the number of neutrons in the nuclei. This should not affect the conduction electrons at all.

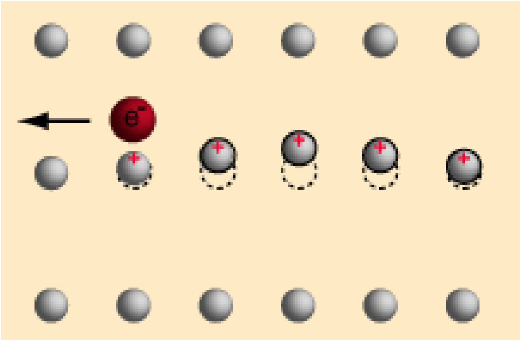
So why do the neutrons change the superconducting temperature?

Unless movement of the atoms are somehow involved in causing the superconductivity.

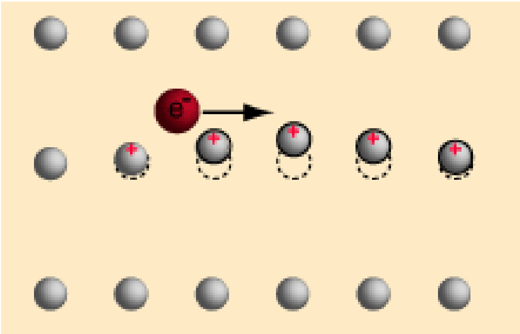
More neutrons means more mass. This would result in slower movement of atoms.

The provided an important clue: Lattice vibration is known to scatter electrons and cause resistance.

When a electron moves in a metal, it can attract the positive ions and bring them closer.



Another electron may then get attracted to the displaced ions.



http://hyperphysics.phy-astr.gsu.edu/hbase/solids/coop.html

The idea of electron attraction mediated by phonons (vibration) was developed around 1950, notably by Frölich and Bardeen.

In 1957, Bardeen, Cooper and Schrieffer (BCS) used quantum mechanics to work out how electrons could form stable pairs. The BCS theory is able to explain many of the features in superconductivity.

The electrons pairs are now called Cooper pairs. These are now widely accepted to be the cause of superconductivity.

We shall see that Cooper pairs can also explain superfluidity in liquid helium-3, which are not bosons.

As an example of a prediction by the BCS theory, recall the behaviour of heat capacity in a superconductor:

$$C_v = a\,\mathrm{exp}\left(-\frac{\Delta}{k_B T}\right)$$

This looks like the Boltzmann factor, in which $\Delta$ is the energy between two levels. The BCS theory explains this as the energy needed to break up the Cooper pair.

This energy is now called the energy gap. It can be obtained directly from a heat capacity measurement by fitting the above formula.

BCS theory predicts that the energy gap and the transition temperature are related by:

$$2\Delta = 3.52 k_B T_c.$$

Rearrange the relation gives this ratio:

$$\frac{2\Delta}{k_B T_c} = 3.52.$$

The ratio for measured values are shown here:

| Superconductor | $2\Delta(0)/k_B T_c$ |
|---|---|
| Al | $3.37 \pm 0.1$ |
| Cd | $3.20 \pm 0.1$ |
| Hg | $4.60 \pm 0.1$ |
| In | $3.63 \pm 0.1$ |
| Nb | $3.84 \pm 0.06$ |
| Pb | $4.29 \pm 0.04$ |
| Sn | $3.46 \pm 0.1$ |
| Ta | $3.60 \pm 0.1$ |

Meservey and Schwarz, in Parks (1969) Superconductivity

The ratios are all fairly close to 3.52.

We have seen how a macroscopic wavefunction is able to explain the zero resistance and the Meissner's effect.

However, when London first proposed the idea in 1937, it was not known how a macroscopic wavefunction could exist in a superconductor.

In the case of liquid helium-4, we know it is possible because helium-4 atoms are bosons, which may undergo Bose-Einsten condensation.

Electrons, however, are fermions. Because of the exclusion principle, it is not possible for all of them to condense to the same state to form a macroscopic wavefunction.

We now have the answer in the Cooper pair.

Although an electron has spin 1/2, when two of them form a Cooper pair, the spins cancel. The Cooper pair has spin zero and is, therefore, a boson.

So Bose-Einstein condensation can take place for the Cooper pairs. They can all condense into the ground state to to form a macroscopic wavefunction.

This provides the explanation for London's wavefunction, and completes the basic theory of superconductivity.

Existing applications of superconductivity include:

1. Maglev train.
2. Magnetic Resonance Imaging (MRI)
3. Particle accelerators (e.g. LHC)
4. Detecting weak magnetic field (SQUIDS)

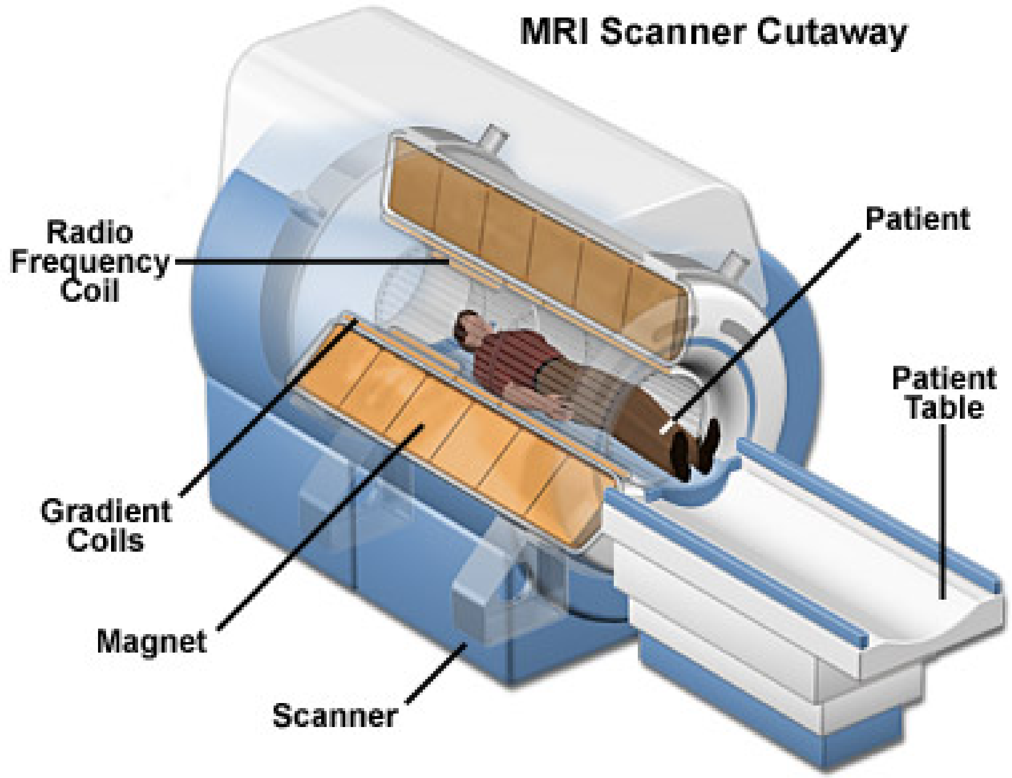
http://www.superconductors.org/uses.htm

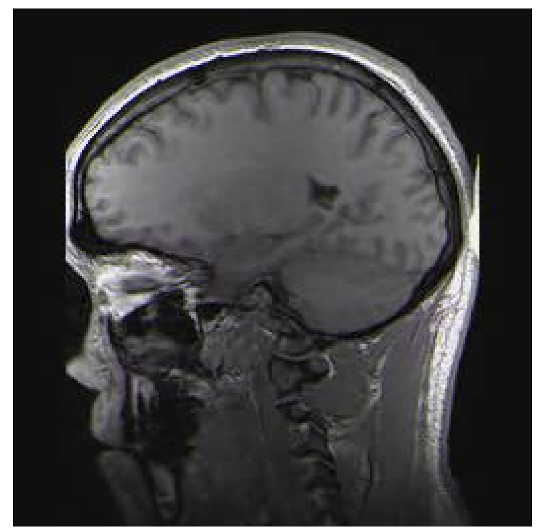
A train can be levitated above its track using powerful, superconducting magnets, so that there is little friction.

One, built in Japan in 2005, travelled at half the speed of sound.

http://en.wikipedia.org/wiki/Maglev_(transport)

MRI requires a very strong magnetic field. This is produced using supercondctors.



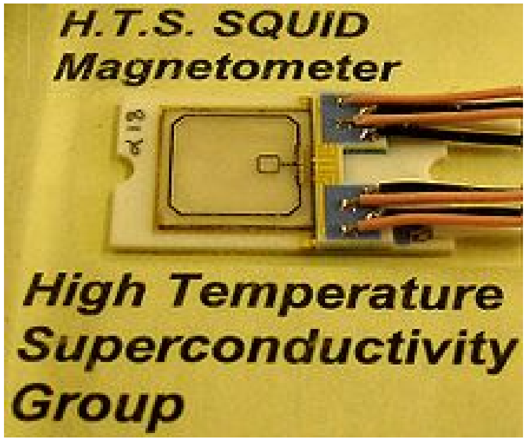
http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/
(and Wikipedia)

Particle accelerators use superconducting magnets and rf cavities to accelerate particles to high energies.

The Large Hadron Collider:

## Detecting weak magnetic field

A superconducting device called SQUID can detect very weak magnetic fields.




(Wikipedia) It is useful for:

- detecting brainwave,
- diagnosing problems in various parts of the human body,
- as an MRI detector,
- oil prospecting,
- earthquake prediction,
- submarine detection, etc.